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Heat-transfer enhancement and pressure loss by surface roughness in turbulent channel flows

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Abstract

A theoretical study of turbulent channel flows was conducted in order to investigate the relationship between the heat-transfer enhancement and the increase in drag by rough surfaces using time–space averaged momentum and energy equations. The results confirmed that the efficiency of heat-transfer surfaces is in general less than unity when the molecular Prandtl number of the working fluid is less than the turbulent Prandtl number. When the molecular Prandtl number is greater than the turbulent Prandtl number, on the other hand, it was shown that the efficiency could be greater than unity as long as the surface roughness is transitional. It is suggested that the breakdown of the Reynolds analogy being experimentally observed for the riblet surface seems to be due to the inhomogeneity of the heat flux over the micro-grooves at the initial stage of thermal boundary-layer development.

1. Introduction

Over the years, many engineering techniques have been devised for enhancing the rate of convective heat-transfer from the wall surface [1,2]. The use of roughness elements is a typical example of this application to increase the heat-transfer coefficient from the flow surface through an increase in turbulent motion. The heat-transfer enhancement is, however, usually accompanied by an increase in drag. From an engineering point of view, it is desirable that the heat-transfer should be enhanced with a minimum drag penalty. A number of studies have been carried out, therefore, to investigate the efficiency of heat-transfer enhancement from rough surfaces. Nunner [3] studied two-dimensional transverse ribs for both the heat-transfer enhancement as well as for the increase in drag. Han et al. [4–6] investigated different shapes of ribs in an effort to identify the range of parameters that give the best heat-transfer efficiency. They suggested that the V-shaped broken ribs with 60° inclination to the flow were the most efficient shape, although the drag increase from these ribs is nearly twice as much as the corresponding heat-transfer enhancement. Other experimental results [7–9] also showed that there are greater drag increases by the roughness elements than the associated heat-transfer enhancement. It is widely believed that the drag increase over rough surfaces is much greater than the heat-transfer enhancement when air is used as a working fluid.

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Nomenclature

A	surface area
a	blockage factor
b	channel half width
c	ratio of pressure losses between rough and smooth channels
c_p	specific heat
E	$E \equiv 1 - \frac{1}{\Gamma}\int_0^y \overline{U}\,dy$
f_{pro}	drag force per unit span acting on roughness elements
h	heat-transfer coefficient
k	roughness height
P	pressure
$\bar{P}$	time-averaged pressure
p	pressure fluctuation
$\overline{\left(\frac{dP}{dx}\right)}$	mean pressure gradient in flow direction
Pr	Prandtl number
Pr_t	turbulent Prandtl number
Pr_{eff}	effective Prandtl number
$\dot{q}$	heat flux from wall
$\dot{q}_{pro}$	heat flux from roughness surface
R_d	contribution of roughness surface to whole drag
R_h	contribution of roughness surface to whole heat transfer
T	temperature
T^+	non-dimensional temperature
$\overline{T}$	mean temperature
$\overline{\left(\frac{dT}{dx}\right)}$	mean temperature gradient in flow direction
ΔT	temperature difference between wall surface and channel centre
U	longitudinal velocity
$\overline{U}$	mean longitudinal velocity
u	longitudinal velocity fluctuation
u^*	friction velocity
u^+	non-dimensional velocity, $u^+ = u/u^*$
V	normal velocity
v	normal velocity fluctuation
W	spanwise velocity
w	spanwise velocity fluctuation
x	coordinate in flow direction
y	coordinate normal to the wall
y^+	non-dimensional distance, $y^+ = y \cdot u^*/\nu$
z	coordinate in spanwise direction

Greek symbols

α	thermal diffusivity
δ	momentum boundary-layer thickness
δ_T	thermal boundary-layer thickness
ε_t	eddy viscosity
Γ	volumetric flow rate per unit width of the channel
η	heat-transfer efficiency of rough surface
λ	thermal conductivity
μ	viscosity
ν	kinematic viscosity
θ	temperature fluctuation
ρ	density
τ	time

Superscripts

$'$	rough surface
–	time averaged value
=	time-space averaged value
+	non-dimensional values in viscous units

Kolar [10] investigated the screw thread-type roughness in a pipe by using different working fluids, and found that the heat-transfer coefficient of the roughness elements increases with the Prandtl number. Dipprey and Sabersky [11] studied the granular roughness over the pipe surface and showed that the heat-transfer enhancement exceeds the associated increase in the pressure drop. The improvement in the heat-transfer efficiency was, however, limited to the cases where the Prandtl number is greater than 3 and the roughness surfaces are transitional. Webb et al. [12] and Cheng et al. [13–15] also obtained similar results indicating a possibility of enhancing the heat transfer from rough surfaces with a minimum increase in drag.

The main purpose of this study is to investigate the heat-transfer efficiency of roughness elements of arbitrary shape, by paying particular attention to the effects of the molecular Prandtl number on heat transfer. Time–space averaged momentum and energy equations are used in the present theoretical work, in order to compare the heat-transfer efficiency of rough surfaces with that of a smooth surface in turbulent channel flows. Recent studies on riblet surface demonstrate that the heat-transfer coefficient can be increased more than the increase in the associated skin-friction drag, indicating an apparent breakdown of the Reynolds analogy. These results were obtained initially by Walsh and Weinstein [16], followed by Choi [17] and Choi and Orchard [18]. There is also a numerical study [19] indicating that the heat transfer efficiency may be improved with an increase in the Prandtl number within the drag- reduction regime of riblets. Possible mechanisms of heat-transfer enhancement over the riblet surface will also be discussed in the present study in a context of improved heat-transfer efficiency of longitudinal micro-grooves.

2. Basic equations

A fully developed turbulent flow in a two-dimensional channel with a constant heat flux from the wall, as shown in Fig. 1(a), is considered in this study. Three-dimensional, arbitrary-shaped roughness elements are placed on the bottom surface of the channel at $y = 0$ as shown in Fig. 1(b). It is assumed that the heat flux from the bottom surface of the channel is the same irrespective of the surface roughness. The flow rate is also assumed to be the same for rough- and smooth-surface channels. The streamwise component of the Navier–Stokes equation and the energy equation, which will be used in the present analysis, are given by

$$\rho\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+W\frac{\partial U}{\partial z}\right) = -\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right) \tag{1}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}+W\frac{\partial T}{\partial z}\right) = \lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{2}$$

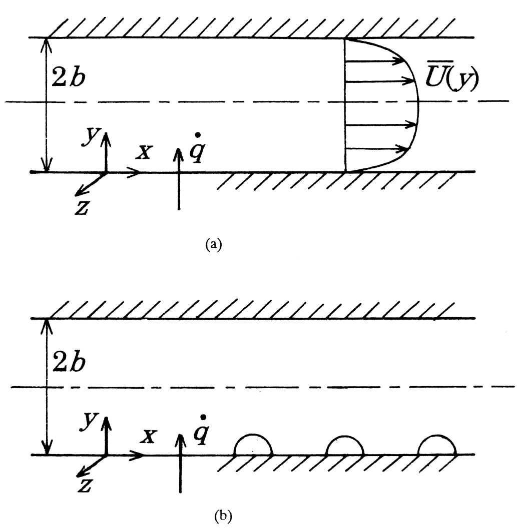

Fig. 1. Coordinate system of turbulent channel flow with a constant heat flux from the wall surface. (a) Smooth wall; (b) rough wall.

2.1. Channel with smooth surface

In order to derive the basic equations for a smooth-surface channel, we first take a time average of Eqs. (1) and (2). During this process, usual Reynolds decomposition is applied to all the velocity components U, V and W, and to the pressure P as

$$U=\overline{U}+u, \qquad V=v, \qquad W=w, \qquad P=\bar{P}+p$$

where $\overline{U}$ and $\bar{P}$ indicate the time-averaged velocity and pressure, respectively. We only deal with a fully-developed temperature field with constant heat flux from the wall. In this case, the time-averaged temperature profile becomes linear in x [20]. Therefore, the temperature can be decomposed into

$$T=\overline{T}_0+\overline{\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)}x+\theta \tag{3}$$

where, $\overline{T}_0 \equiv \overline{T}_0(y)$ is the time-averaged temperature at $x=0$, and $\overline{(\frac{\mathrm{d}T}{\mathrm{d}x})}$ is the constant temperature gradient. In the rest of the analysis, however, $\overline{T}_0$ is written as $\overline{T}$ for simplicity. Due to the constant heat-flux condition at the wall, the mean temperature gradient in the streamwise direction is given by

$$\overline{\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)}=\frac{\dot{q}}{\rho c_p \Gamma}$$

Integrating Eqs. (1) and (2) in y direction gives the following equations after time-averaging operation,

$$-\overline{\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)}(b-y)=\mu\frac{\mathrm{d}\overline{U}}{\mathrm{d}y}-\rho\overline{uv}=\rho(\nu+\varepsilon_{\mathrm{t}})\frac{\mathrm{d}\overline{U}}{\mathrm{d}y} \tag{4}$$

$$\dot{q}\left(1-\frac{1}{\Gamma}\int_0^y \overline{U}\,\mathrm{d}y\right)=\lambda\frac{\mathrm{d}\overline{T}}{\mathrm{d}y}-\frac{\partial}{\partial y}\left(\rho c_p\overline{v\theta}\right) = \rho c_p\left(\alpha+\frac{\varepsilon_{\mathrm{t}}}{Pr_{\mathrm{t}}}\right)\frac{\mathrm{d}\overline{T}}{\mathrm{d}y} \tag{5}$$

Here, the turbulent transport terms are given in terms of the eddy viscosity ε_{t} and the turbulent Prandtl number Pr_{t}.

2.2. Channel with rough surface

Velocities over a rough surface can be given by

$$U=\overline{\overline{U}}'+u' \qquad V=v' \qquad W=w' \tag{6}$$

where $\overline{\overline{U}}'$ is the velocity averaged in x–z plane and time, and u', v' and w' are the velocity fluctuations in x-, y- and z-directions, respectively. Similarly, the pressure gradient can be given by

$$\frac{\partial P}{\partial x} = \overline{\overline{\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)'}} + \frac{\partial P'}{\partial x} \tag{7}$$

After substituting Eqs. (6) and (7), Eq. (1) is integrated once in x–z plane and time, followed by an integration in y direction. The left-hand side of the Navier–Stokes equation (1) will then take the following form using the continuity equation,

$$\frac{1}{\tau}\int_t^{t+\tau} \mathrm{d}t \int_0^y \mathrm{d}y \frac{1}{A}\iint_{xz} \mathrm{d}x\,\mathrm{d}z \left\{\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z}\right)\right\} = a \cdot \rho\overline{\overline{u'v'}} \tag{8}$$

where τ and A are time and surface area, respectively where the integrations are performed. The blockage of the channel flow by roughness elements is given by $a \equiv a(y)$, $0 \le a \le 1$.

Using Gauss's integral theorem, the right-hand side of the Navier–Stokes equation (1) is given by a surface integral of the stresses over the x–z plane as

$$\frac{1}{\tau}\int_t^{t+\tau} \mathrm{d}t \int_0^y \mathrm{d}y \frac{1}{A}\iint_{xz} \mathrm{d}x\,\mathrm{d}z \left\{-\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right)\right\} = -\overline{\overline{\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)'}}(y-b) + a \cdot \mu\frac{\mathrm{d}\overline{\overline{U}}'}{\mathrm{d}y} + f_{\mathrm{pro}} \tag{9}$$

where f_{pro} indicates the drag on the roughness surface above the integral x–z plane. From Eqs. (8) and (9), time–space averaged momentum equation can be obtained as follows

$$-\overline{\overline{\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)'}}(b-y) = \rho a\left(\nu + \varepsilon_{\mathrm{t}}'\right)\frac{\mathrm{d}\overline{\overline{U}}'}{\mathrm{d}y} + f_{\mathrm{pro}} \tag{10}$$

Similar to Eq. (3), the temperature for rough-surface channel is given by

$$T = \overline{\overline{T}}' + \overline{\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)}x + \theta' \tag{11}$$

The average temperature gradient $\overline{(\frac{\mathrm{d}T}{\mathrm{d}x})}$ in this equation is the same as that in smooth-surface channel, because the flow rate Γ and the flux $\dot{q}$ from the bottom surface of the channel are the same irrespective of the surface roughness. Eq. (2) is averaged in x–z plane and time after substituting Eq. (11), which is then be integrated in y direction. Using Gauss's integral theorem and the Continuity equation, the following equation can be derived

$$\rho c_p\overline{\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)}\int_0^y a\overline{\overline{U}}'\,\mathrm{d}y + a \cdot \rho c_p\overline{\overline{v'\theta'}} = \dot{q} - \dot{q}_{\mathrm{pro}} + a \cdot \lambda\frac{\mathrm{d}\overline{\overline{T}}'}{\mathrm{d}y} \tag{12}$$

which can be rearranged as follows:

$$\dot{q}\left(1 - \frac{1}{\Gamma}\int_0^y a\overline{\overline{U}}'\,\mathrm{d}y\right) = a \cdot \rho c_p\left(\alpha + \frac{\varepsilon_{\mathrm{t}}'}{Pr_{\mathrm{t}}'}\right)\frac{\mathrm{d}\overline{\overline{T}}'}{\mathrm{d}y} + \dot{q}_{\mathrm{pro}} \tag{13}$$

2.3. Non-dimensional equations

Before discussing the relationship between the heat-transfer enhancement and the pressure loss in a channel, the momentum equations (4) and (10) and the energy equations (5) and (13) are non-dimensionalised using wall variables. The pressure gradient along the channel can be given in terms of the friction velocity u^* for the smooth surface as

$$-b\overline{\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)} = \rho u^{*2}, \qquad -b\overline{\overline{\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)'}} = c \cdot \rho u^{*2} \tag{14}$$

where c is the pressure-loss ratio between rough- and smooth-surface channels. We have

$$1 - \frac{y^+}{b^+} = \left(1 + \frac{\varepsilon_{\mathrm{t}}}{\nu}\right)\frac{\mathrm{d}\overline{U}^+}{\mathrm{d}y^+} \tag{15}$$

$$E = \left(\frac{1}{Pr} + \frac{\varepsilon_{\mathrm{t}}}{Pr_{\mathrm{t}}\nu}\right)\frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+} \tag{16}$$

for smooth-surface channel, and

$$\left(1 - \frac{y^+}{b^+}\right)(1 - R_{\mathrm{d}}) = \frac{a}{c}\left(1 + \frac{\varepsilon_{\mathrm{t}}'}{\nu}\right)\frac{\mathrm{d}\overline{\overline{U}}'^+}{\mathrm{d}y^+}, \quad R_{\mathrm{d}} = \frac{f_{\mathrm{pro}}}{-\overline{\overline{\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)'}}(b-y)} \tag{17}$$

$$E'(1 - R_{\mathrm{h}}) = a\left(\frac{1}{Pr} + \frac{\varepsilon_{\mathrm{t}}'}{Pr_{\mathrm{t}}'\nu}\right)\frac{\mathrm{d}\overline{\overline{T}}'^+}{\mathrm{d}y^+}, \quad R_{\mathrm{h}} = \frac{\dot{q}_{\mathrm{pro}}}{\dot{q}E'} \tag{18}$$

for rough-surface channel. Here, the non-dimensional temperature is defined by

$$\overline{T}^+ = \frac{\rho c_p u^* \overline{T}}{\dot{q}} \tag{19}$$

Here, R_d and R_h are the contribution of rough surfaces to drag and heat flux, respectively for those roughness elements above the x–z integral plane. The heat-transfer coefficients h and h' for smooth and rough surfaces, respectively are non-dimensionalised as follows:

$$\begin{aligned} \frac{h}{\rho c_p u^*} &= \frac{1}{\Delta \overline{T}^+} = \frac{1}{\int_0^{b^+} \frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+}\,\mathrm{d}y^+}, \\ \frac{h'}{\rho c_p u^*} &= \frac{1}{\Delta \overline{\overline{T}}'^{\,+}} = \frac{1}{\int_0^{b^+} \frac{\mathrm{d}\overline{\overline{T}}'^{\,+}}{\mathrm{d}y^+}\,\mathrm{d}y^+} \end{aligned} \tag{20}$$

3. Relationship between heat-transfer enhancement and pressure loss

3.1. Drag-increasing surfaces

In this section, we focus our discussion on determining whether or not it is possible for the heat-transfer efficiency to be greater than unity for some roughness elements. Here, the heat- transfer efficiency η is defined as the ratio of the heat-transfer enhancement to the associated increase in drag. In other words, we will examine whether or not the heat-transfer enhancement by rough surfaces, as given by h'/h, can be greater than the increase in pressure loss given by the factor c. In order to do this, the following quantity is examined.

$$\Delta\overline{T}^+ - c\Delta\overline{\overline{T}}'^{\,+} = \int_0^{b^+}\left(\frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+} - c\frac{\mathrm{d}\overline{\overline{T}}'^{\,+}}{\mathrm{d}y^+}\right)\mathrm{d}y^+ \tag{21}$$

It is clear that the heat-transfer efficiency will be greater than unity if the quantity given by Eq. (21) is positive. First and second term of the integrand in Eq. (21) can be rewritten as follows using Eqs. (15)–(18).

$$\frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+} = \frac{E}{(1-y^+/b^+)}\frac{Pr_t(1+\varepsilon_t/\nu)}{(Pr_t/Pr+\varepsilon_t/\nu)}\frac{\mathrm{d}\overline{U}^+}{\mathrm{d}y^+} \tag{22}$$

$$c\frac{\mathrm{d}\overline{\overline{T}}'^{+}}{\mathrm{d}y^+} = \frac{E'}{(1-y^+/b^+)}\frac{(1-R_h)}{(1-R_d)}\frac{Pr_t'(1+\varepsilon_t/\nu)}{\left(Pr_t'/Pr+\varepsilon_t/\nu\right)}\frac{\mathrm{d}\overline{\overline{U}}'^{\,+}}{\mathrm{d}y^+} \tag{23}$$

Therefore, we only need to compare the quantities in Eqs. (22) and (23) to examine the heat-transfer efficiency of rough surfaces.

A majority of studies on channel and boundary layer flows suggest that the turbulent Prandtl number Pr_t takes a constant value of about 0.85 for any fluids except very close to the wall surface [21–25]. It is also shown by Pimenta et al. [26] that the turbulent Prandtl number for rough surfaces is nearly equal to that of smooth surfaces, i.e. $Pr_t = Pr_t'$ in the logarithmic region, which is supported by recent numerical studies [8,9]. We firstly consider the case where the molecular Prandtl number is equal or less than the turbulent Prandtl number, i.e. $Pr \le Pr_t = 0.85$.

Since the flow rate near a rough surface is less than that over a smooth wall, we have

$$E' > E \tag{24}$$

We also know that the eddy viscosity over a rough surface is greater than that over a smooth surface, i.e. $\varepsilon_t' > \varepsilon_t$. Using these relationships we have the following inequality for $y^+ > k^+$, considering that $R_h = R_d = 0$

$$\begin{aligned} &\frac{E'}{(1-y^+/b^+)}\frac{Pr_t'\left(1+\varepsilon_t'/\nu\right)}{\left(Pr_t'/Pr+\varepsilon_t'/\nu\right)} \\ &\quad > \frac{E}{(1-y^+/b^+)}\frac{Pr_t(1+\varepsilon_t/\nu)}{(Pr_t/Pr+\varepsilon_t/\nu)} \end{aligned} \tag{25}$$

For the region very close to the wall ($y^+ < k^+$) we assume that the contribution of roughness elements to the drag, R_d is greater than that to the heat-transfer, R_h, that is

$$(1-R_h) > (1-R_d) \tag{26}$$

This is because large form drag, which is unanalogous to heat transfer, acts on the roughness surface [27]. In fact, R_d is much greater than R_h for hemispherical roughness elements according to the results by Hosni et al. [8]. They showed, for example, that $R_h \approx 0.3$ and $R_d \approx 0.6$ for hemispherical roughness elements of the pitch-to-diameter ratio of 4, making the value of the left-hand side of Eq. (26) roughly twice as that of the right-hand side. Using Eqs. (24) and (26), a similar relationship to (25) can be derived for $y^+ < k^+$

$$\begin{aligned} &\frac{E'}{(1-y^+/b^+)}\frac{(1-R_h)}{(1-R_d)}\frac{Pr_t'\left(1+\varepsilon_t'/\nu\right)}{\left(Pr_t'/Pr+\varepsilon_t'/\nu\right)} \\ &\quad > \frac{E}{(1-y^+/b^+)}\frac{Pr_t(1+\varepsilon_t/\nu)}{(Pr_t/Pr+\varepsilon_t/\nu)} \end{aligned} \tag{27}$$

This relationship does not hold, however, for $Pr_t' < Pr_t$ or $Pr > Pr_t$ when

$$\frac{Pr_t(1+\varepsilon_t/\nu)}{(Pr_t/Pr+\varepsilon_t/\nu)} > \frac{Pr_t'\left(1+\varepsilon_t'/\nu\right)}{\left(Pr_t'/Pr+\varepsilon_t'/\nu\right)}.$$

which will be considered later.

The following inequality can be derived from Eqs. (25) and (27),

$$\int_0^{b^+}\left(\frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+}-c\frac{\mathrm{d}\overline{\overline{T}}'^+}{\mathrm{d}y^+}\right)\mathrm{d}y^+ < \int_0^{b^+} g^+\left(\frac{\mathrm{d}\overline{U}^+}{\mathrm{d}y^+}-\frac{\mathrm{d}\overline{\overline{U}}'^+}{\mathrm{d}y^+}\right)\mathrm{d}y^+ = g^+(b^+)\left\{\overline{U}^+(b^+)-\overline{\overline{U}}'^+(b^+)\right\}+\int_0^{b^+}\frac{\mathrm{d}g^+}{\mathrm{d}y^+}\left\{\overline{\overline{U}}'^+-\overline{U}^+\right\}\mathrm{d}y^+ \tag{28}$$

where

$$g^+(y^+)=\frac{E}{(1-y^+/b^+)}Pr_{\mathrm{eff}},\qquad Pr_{\mathrm{eff}}=\frac{Pr_{\mathrm{t}}(1+\varepsilon_{\mathrm{t}}/\nu)}{(Pr_{\mathrm{t}}/Pr+\varepsilon_{\mathrm{t}}/\nu)} \tag{29}$$

The quantity $(\mathrm{d}g^+/\mathrm{d}y^+)$ in Eq. (28) is a positive, monotonically decreasing function of y^+, referring to the cal culated result of Pr_{eff} [21], and the velocity $\overline{\overline{U}}'^+$ over a rough surface is less than the velocity $\overline{U}^+$ over a smooth surface except near the centre of channel [28]. Therefore, the last integral in Eq. (28) becomes negative, while the inequality $\overline{U}^+(b^+) < \overline{\overline{U}}'^+(b^+)$ holds for the first term of Eq. (28). The following relationship can be obtained as the result.

$$\int_0^{b^+}\left(\frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+}-c\frac{\mathrm{d}\overline{\overline{T}}'^+}{\mathrm{d}y^+}\right)\mathrm{d}y^+ < 0 \tag{30}$$

This suggests that the amount of heat-transfer enhancement by roughness elements is less than the associated increase in pressure loss when $Pr \le Pr_{\mathrm{t}} = 0.85$, unless Pr'_{t} becomes very small at $y^+ < k^+$.

Since the effective Prandtl number Pr_{eff} represents the ratio between the effective eddy diffusivity of momentum and that of heat transfer, the momentum transport is more effective than the heat transfer for flows with large Pr_{eff}. From Eq. (29), however, it can be shown that the increase in eddy viscosity from ε_{t} to $\varepsilon'_{\mathrm{t}}$ by surface roughness leads to $Pr_{\mathrm{eff}} \le Pr'_{\mathrm{eff}}$ when $Pr \le Pr_{\mathrm{t}}$. Hence, the enhancement of heat transfer from rough surfaces is not as effective as the increase in the momentum transport when $Pr \le Pr_{\mathrm{t}}$.

3.2. Conditions for $\eta > 1$

We can see from Eq. (29) that the increase in eddy viscosity by surface roughness will improve the relative effectiveness in transporting heat to momentum when $Pr \gg Pr_{\mathrm{t}}$, suggesting that the heat-transfer efficiency may be increased by roughness elements when Pr becomes greater than Pr_{t}. In order to investigate this possibility, we consider the case where the molecular Prandtl number is increased by a reduction in thermal diffusivity, i.e. $Pr \to \infty$. Using Eqs. (16) and (18), Eq. (21) can be written as

$$\int_0^{b^+}\left(\frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+}-c\frac{\mathrm{d}\overline{\overline{T}}'^+}{\mathrm{d}y^+}\right)\mathrm{d}y^+ \cong \int_0^{b^+}\left\{E\frac{Pr_{\mathrm{t}}\nu}{\varepsilon_{\mathrm{t}}}-\frac{c}{a}E'(1-R_{\mathrm{h}})\frac{Pr_{\mathrm{t}}\nu}{\varepsilon'_{\mathrm{t}}}\right\}\mathrm{d}y^+ \tag{31}$$

where we assume that $Pr_{\mathrm{t}} = Pr'_{\mathrm{t}}$ for simplicity.

Firstly, the integral in Eq. (31) is evaluated for $y^+ > k^+$, where the mixing length over a rough surface has in general a similar value to that over a smooth surface [28]. This means that the ratio of eddy viscosities, $(\varepsilon'_{\mathrm{t}}/\varepsilon_{\mathrm{t}})$ between the rough and smooth surfaces is equal to that of the ratio of the friction velocities, $\sqrt{c}$. Since $R_{\mathrm{h}} = 0$ and $E' > E$ for $y^+ > k^+$, Eq. (31) with Eq. (24) gives

$$\int_{k^+}^{b^+}\left(\frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+}-c\frac{\mathrm{d}\overline{\overline{T}}'^+}{\mathrm{d}y^+}\right)\mathrm{d}y^+ < \int_{k^+}^{b^+} Pr_{\mathrm{t}}\nu E\frac{1-\sqrt{c}}{\varepsilon_{\mathrm{t}}}\mathrm{d}y^+ < 0 \tag{32}$$

This indicates that the integral takes a negative value, considering that $c > 1$.

Next, the integral in Eq. (31) is evaluated for $y^+ \le k^+$, where we again assume that $\varepsilon'_{\mathrm{t}}/\varepsilon_{\mathrm{t}} = \sqrt{c}$. Since $E' \approx E \approx 1$ for small y, Eq. (31) becomes

$$\int_0^{k^+}\left(\frac{\mathrm{d}\overline{T}^+}{\mathrm{d}y^+}-c\frac{\mathrm{d}\overline{\overline{T}}'^+}{\mathrm{d}y^+}\right)\mathrm{d}y^+ = \int_0^{k^+} Pr_{\mathrm{t}}\nu E\frac{a-\sqrt{c}(1-R_{\mathrm{h}})}{a\varepsilon_{\mathrm{t}}}\mathrm{d}y^+ \tag{33}$$

If the integral in Eq. (33) is positive and is large enough to compensate for the negative term in Eq. (32) for $y^+ > k^+$, the heat-transfer enhancement can be greater than the associated drag increase. To guarantee that the integral in Eq. (33) is positive, we require that:

1. blockage of channel flow by roughness elements should not be large (a should not be small);
2. increase in pressure loss by surface roughness should not be large (c should not be large);
3. contribution of roughness elements to heat transfer R_{h} must be large.

According to the results by Hosni et al. [8], the integral value in Eq. (33) is positive for semispherical roughness with pitch-to-diameter-ratio of 4 when $a \approx 0.95$, $c \approx 1.25$ and $R_{\mathrm{h}} \approx 0.3$. It becomes negative, however, even when $Pr \to \infty$ if there is no contribution of roughness elements to heat-transfer (i.e., $R_{\mathrm{h}} = 0$).

In order to quantify the effect of Prandtl number on the relative increase in heat transfer to drag, the heat-transfer efficiency η of roughness elements is estimated for identical roughness elements studied by Hosni et al. [8] for air. Here, the heat-transfer coefficients are obtained by integrating the difference in temperature in Eqs. (16) and (18) for smooth and rough surfaces, respectively. The non-dimensional roughness height k^+ of semispherical elements being studied with pitch-to-diameter ratio of 4 are 20 and 80, respectively. It is assumed that $\varepsilon_t'/\varepsilon_t = \sqrt{c}$ ($c = 1.25$ at $k^+ = 20$; $c = 1.85$ at $k^+ = 80$) and $Pr_t' = Pr_t = 0.85$ across the channel. The use of the empirical formula suggested in Kays [21] for the turbulent Prandtl number instead of using a constant value only made a small difference in the estimated value of η. The distribution of R_h over the roughness elements was given from the local heat-transfer coefficient obtained by Hosni et al. [8]. Fig. 2 shows the calculated results for smaller roughness elements ($k^+ = 20$), indicating that the heat-transfer efficiency η is greater than unity when $Pr > 5$. For larger roughness elements ($k^+ = 80$), however, the heat-transfer efficiency does not seem to exceed unity for any Prandtl numbers. These findings agree well with the experimental results by Dipprey and Sabersky [11] and with the empirical formula by Webb et al. [12], showing that η can be greater than unity when the surface is transitionally rough ($5 < k^+ < 70$) and the Prandtl number is large. When the roughness height exceeds $k^+ = 70$, however, η may never be greater than unity.

3.3. Heat-transfer over riblets surface

Our discussions so far have been centred on the heat-transfer efficiency of the roughness elements that increase the drag in turbulent channel flows. It has been known, however, that a surface with longitudinal micro-grooves, called riblets, can reduce the skin-friction drag of turbulent boundary-layer and channel flows by up to 10% when their size is optimised [29] to approximately the thickness of the viscous sublayer ($k^+ \approx 15$). Recently, Walsh and Weinstein [16], Choi [17] and Choi and Orchard [18] observed that the heat-transfer coefficient can be increased by a riblet surface at the same time as it reduces the skin-friction drag. This apparently suggests a breakdown of the Reynolds analogy where the heat-transfer efficiency of a riblet surface can be greater than that of a smooth surface. In this section, the conditions for the heat-transfer enhancement by drag-reducing riblets in turbulent channel flows are briefly considered.

We consider the following quantity

$$\Delta\overline{T}^+ - \Delta\overline{\overline{T}}'^+ = \int_0^{b^+} \left\{ \frac{E}{(1/Pr + \varepsilon_t/Pr_t)} - \frac{E'(1-R_h)}{a\left(1/Pr + \varepsilon_t'/Pr_t'\right)} \right\} \mathrm{d}y^+ \quad (34)$$

where $\Delta\overline{T}^+$ is the temperature difference between the smooth wall and the channel centre, while $\Delta\overline{\overline{T}}'^+$ is the temperature difference between the riblet wall and the channel centre. If the quantity in Eq. (34) takes a positive value, the heat-transfer coefficient of a riblet surface is greater than that of a smooth surface. Here, we assume that $E \approx E'$ since the velocity profile over a riblet surface is not significantly different from that over a smooth surface [18].

Fig. 3 shows the logarithmic temperature profiles of thermal boundary layers obtained by Choi and Orchard [18], indicating that the temperature difference $\Delta\overline{\overline{T}}'^+$ is smaller over the riblet surface as compared with the temperature difference $\Delta\overline{T}^+$ over a smooth surface. This clearly demonstrates that there is a corresponding increase in the heat-transfer coefficient by

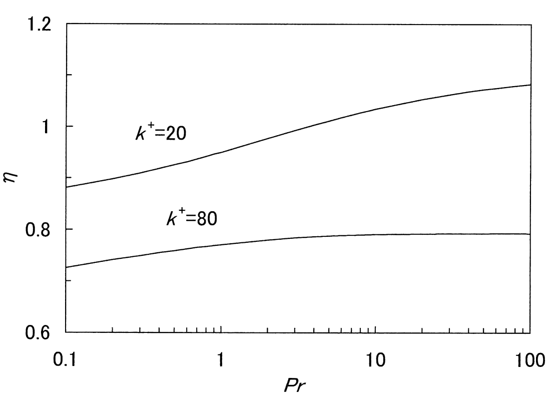

Fig. 2. Heat-transfer efficiency of rough surfaces as a function of the Prandtl number.

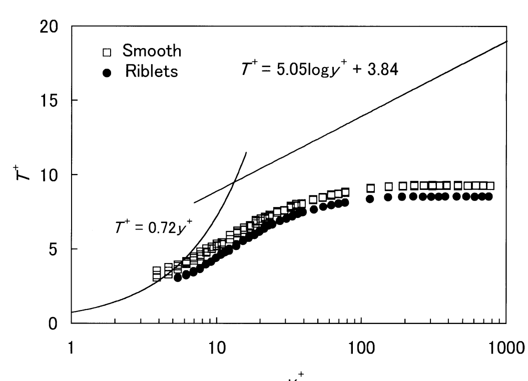

Fig. 3. Logarithmic temperature profiles over smooth and riblet surfaces.

riblets. It should be noted that boundary layer is heated only from the beginning of the riblet section in this experiment, therefore, the thermal boundary layer is not yet fully developed. The thickness of thermal boundary layer is about $\delta_T^+ = 300$, which is much thinner than the fully developed, momentum boundary layer ($\delta^+ = 800$), with a total temperature difference of $\Delta\overline{\overline{T}}'^{+} = 8$. Fig. 3 also indicates that nearly one third of the total temperature difference occurs below the peaks of riblets, which are located at $y^+ = 2$. In this case, we cannot neglect the effect of roughness surface on the heat transfer, i.e. $(1 - R_h)/a$ in Eq. (34), even though the size of riblets is quite small, approximately $k^+ = 13$. In the figure, the downward shift of temperature profile over a riblet surface is already significant at $y^+ = 5$ as compared to that over the smooth surface, indicating a reduced thermal resistance for the riblet surface near the wall. This suggests that $(1 - R_h)/a$ in Eq. (34) is indeed smaller than unity, therefore, the contribution of riblets to heat- transfer R_h should be more than the ratio of roughness surface area $(1 - a)$. It is observed that the downward shift of mean temperature profile near the riblet wall seems to be influencing the entire region of the boundary layer by reducing the total temperature difference. Since $\varepsilon_t > \varepsilon_t'$ and $(1 - R_h)/a = 1$ away from the riblet surface, the integrand in Eq. (34) is negative. It is expected that the heat-transfer coefficient over the heated riblet surface would be reduced with an increase in thermal boundary layer thickness as it develops downstream.

4. Conclusions

The heat-transfer efficiency of rough surfaces, as defined by the ratio of the heat-transfer enhancement by roughness elements to the increase in drag, was theoretically studied for turbulent channel flows using time-space averaged momentum and energy equations. We found that it would be difficult for the heat-transfer efficiency of rough surfaces to become greater than unity when the molecular Prandtl number of working fluid is smaller than the turbulent Prandtl number. This is due to the fact that the contribution of roughness surfaces to the drag increase is usually greater than that to the heat-transfer enhancement.

When the molecular Prandtl number is greater than the turbulent Prandtl number, however, the heat-transfer efficiency may become greater than unity, since the increase in eddy viscosity over rough surfaces could have a greater effect on the heat transfer than the pressure loss. Our calculation indicates that the heat-transfer efficiency becomes greater than unity when the molecular Prandtl number is more than 5 for smaller roughness elements ($k^+ = 20$). For larger roughness elements ($k^+ = 80$), however, the heat-transfer efficiency does not seem to exceed unity for any Prandtl numbers.

Some of the drag-reducing riblet surfaces are known to have high heat-transfer efficiency. When the thermal boundary layer over a riblet surface is thinner than the momentum boundary layer, the effect of roughness elements on heat-transfer enhancement becomes significant even if the riblet size is small. This situation may be encountered when the heat flux is introduced from a riblet surface after the momentum boundary layer is fully developed. It seems, therefore, that the breakdown of the Reynolds analogy being experimentally observed for the riblet surface is due to the inhomogeneity of the heat flux over the micro- grooves at an initial stage of thermal boundary-layer development. All of the previous experiments to show a high heat-transfer efficiency of riblet surfaces were conduced in such a configuration.

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